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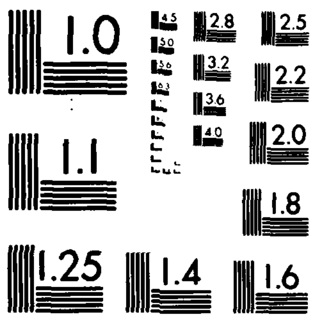
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Pseudosimilar Vertices in a Graph

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PSEUDOSIMILAR VERTICES IN A GRAPH

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Abstract

Dissimilar vertices whose removal leaves isomorphic subgraphs are called pseudosimilar. We construct infinite families of graphs having identity automorphism group, yet every vertex is pseudosimilar to some other vertex. Potential impact on the Reconstruction Conjecture is considered.

We also construct, for each n, graphs containing a subset of vertices of size n which are mutually pseudosimilar. The analogous problem for mutually pseudosimilar edges is introduced.

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1. Introduction

Two vertices u and v of a graph G are called similar if G has an automorphism α that maps u to v. Obviously, whenever u is similar to v, the subgraph G-u is isomorphic to G-v. The isomorphism is trivially obtained by restricting α to G-u. What about the converse - is it equally obvious? Those familiar with the history of the Reconstruction Conjecture know the answer. In the early 60's there was a purported proof of the RC. It relied heavily on the recognition of isomorphic graphs within the deck of vertex deleted subgraphs to identify similar vertices in G. This "proof" was never published because its strategy collapsed when Harary and Palmer [1,2] observed that a graph can have vertices for which G-u $\cong$ G-v and yet u and v are not similar! Let us call u and v removal-similar if G-u $\cong$ G-v. If u and v also happen to be dissimilar, we shall call them pseudosimilar. The graph Harary and Palmer found to be the smallest possible having pseudosimilar vertices is shown in Figure 1.

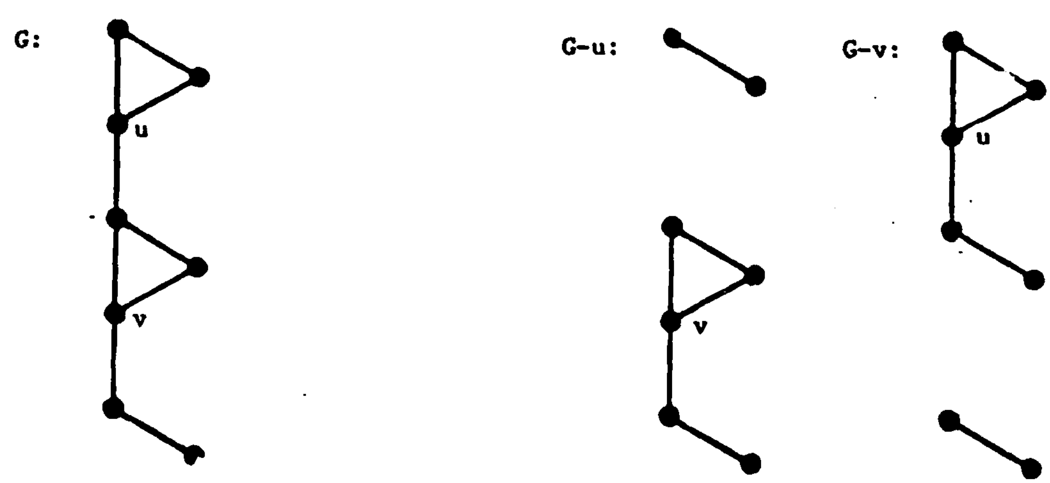
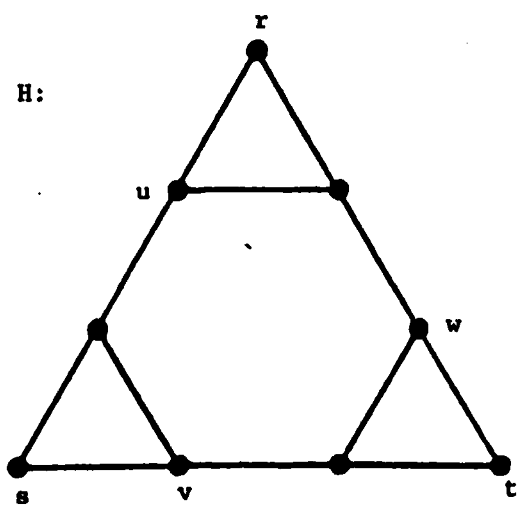


Figure 1: The smallest graph having pseudosimilar vertices.

Removal-similarity is obviously an equivalence relation. This terminology is reminiscent of "removal-cospectral" which was defined in [4]. Indeed, it is clear that removal-similar vertices are, <u>a fortiori</u>, removal-cospectral. On the other hand, pseudosimilarity is neither reflexive nor transitive.

Herndon and Ellzey [3] developed methods to construct removal-cospectral vertices. It was the chance recognition of the Harary-Palmer graph among their examples that led us to recognize that one of their methods was actually producing removal-similar vertices. The procedure starts with a graph H having an automorphism α that permutes some vertices in an orbit of length at least three. An example is shown in Figure 2. Let G be H-w. Now $G-u \cong G-v$ because $G-u = H-w-u$, and so α sends G-u onto $H-\alpha(w)-\alpha(u) = H-v-w = G-v$. That is, u is removal-similar to v in G. If removing w from H has destroyed the similarity of u and v, then u and v are pseudosimilar in G. In our example, G=H-w is the graph of Figure 1. On the other hand, r and s remain similar in H-t, so deleting t does not create pseudosimilar vertices.



$\alpha(u) = w$
$\alpha(w) = v$
$\alpha(v) = u$

$\alpha(r) = t$
$\alpha(t) = s$
$\alpha(s) = r$

<u>Figure 2</u>: An automorphism with orbits of size three.

2. Mutually Pseudosimilar Sets

Appreciating that pseudosimilarity can be built by using the symmetry of a larger graph, we were stimulated to challenge each other to construct graphs having more than just one pseudosimilar pair. A natural question is to decide whether a graph can include k vertices, none of them similar, yet every pair pseudosimilar.

Theorem 1. For each $k \geq 2$, there exist graphs with $k+6\binom{k}{2}$ vertices that include k mutually pseudosimilar vertices.

Proof. We shall construct such graphs. Had the same task been posed for directed graphs, the solution seems to be obvious: choose the transitive tournament T_k in which vertex w_i dominates vertex w_j if and only if $i < j$. Clearly T_k has no automorphisms and yet for any vertex w_i, $T_k - w_i \simeq T_{k-1}$. Our approach is to convert T_k to an undirected graph while maintaining its features. Let H be an arbitrary connected graph, and select two dissimilar nonempty subsets of vertices, A and B. (That is, H has no automorphism mapping A onto B. An easy way to obtain such sets is to choose them to have different sizes. A and B need not be disjoint.) Take three copies of H and join two additional vertices, u and v, to all the vertices of the appropriate subsets A and B as indicated in Figure 3. The graph G so formed has u and v removal-similar. Furthermore, any automorphism sending u to v would carry the left hand copy of H onto the right hand H and it would have to map A onto B. Thus, no such automorphism exists, and so, u and v are pseudosimilar cutpoints. The smallest instance of this construction has $H=K_2$, $|A| = 2$, $|B| = 1$, and yields the ubiquitous graph of Figure 1.

G:

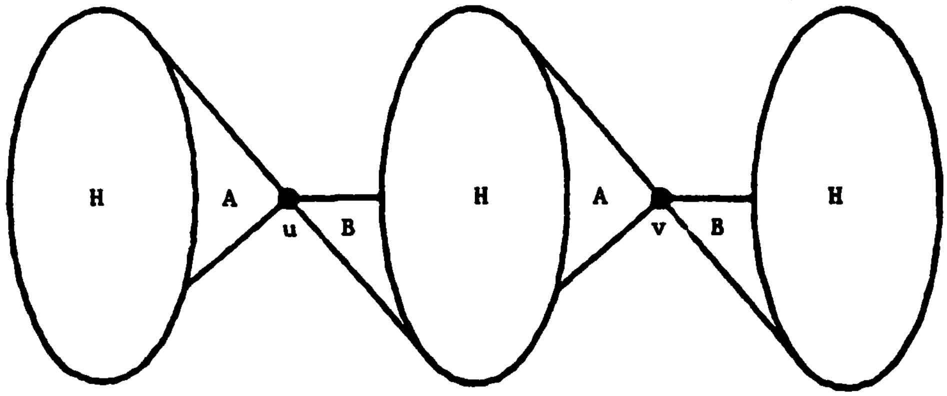


Figure 3: The construction of pseudosimilar cutpoints.

We form a graph $T_k(G,u,v)$ by replacing each arc of T_k by the graph G with u attached at the tail of the arc and v attached at the head. This graph has $k+(|G|-2)\binom{k}{2}$ vertices; the smallest instance has $k+6\binom{k}{2}$ vertices. For example, $T_4(G,u,v)$ is shown in Figure 4, using the graph G of Figure 1. In $T_k(G,u,v)$ each of the k vertices arising from T_k is recognizable as a cutpoint whose removal leaves k-1 copies of H and one large component. Vertex w_i is joined to i-1 of these copies of H at set B and to k-i of them at A. Consequently, all automorphisms of $T_k(G,u,v)$ when restricted to $\{w_1, w_2, \ldots, w_k\}$ must yield the identity, and so these vertices are dissimilar.

Now $T_k(G,u,v)-w_i$, for all i, leaves the same graph. There are k-1 copies of H and the large component resembles $T_{k-1}(G,u,v)$ except it has two extra copies of H attached at each remaining w_j. One of these is attached at A and the other is joined at B. Thus, these k subgraphs are all isomorphic, and so the k vertices are mutually pseudosimilar. ∎

$T_4(G,u,v)$:

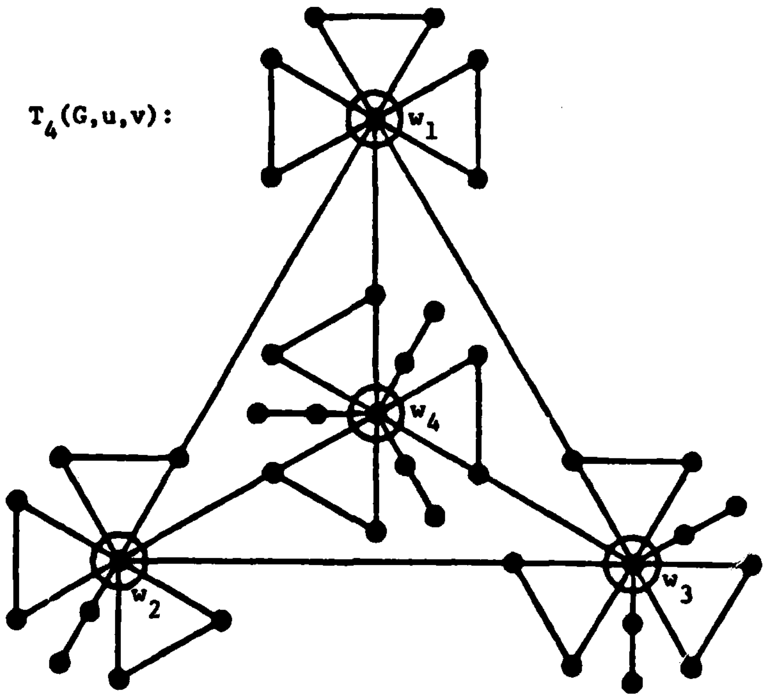
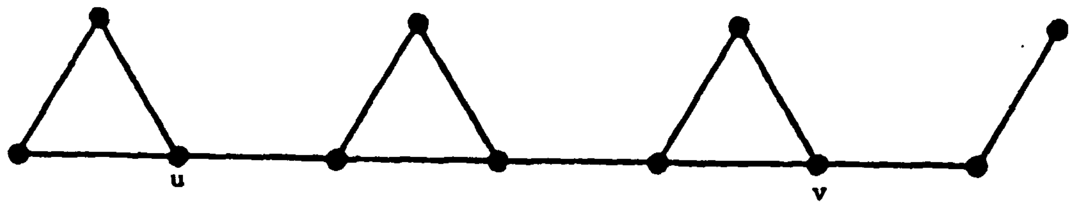


Figure 4: A graph with four mutually pseudosimilar vertices.

A comment is in order. The fact that u and v form a pseudosimilar pair in G might lead one to suspect that any graph with a pseudosimilar pair of vertices can be used in this construction. There are two flaws with this idea. First, if u and v are not cutpoints, it is difficult to recognize the vertices w_i in $T_k(G,u,v)$. These vertices may still be pseudosimilar, but, if so, it is much more difficult to demonstrate the fact. The second flaw is fatal. When the pseudosimilar cutpoints in Figure 5 are used to construct $T_k(G',u,v)$, we find that $T_k(G',u,v)-w_i \not\cong T_k(G',u,v) - w_j$ for $k \geq 3$. Why does G of Figure 1 work while G' of Figure 5 fails? The difference is that the isomorphism sending G-u onto G-v also maps v to u. This property was needed implicitly in the construction of $T_k(G,u,v)$. In contrast the isomorphism

sending G'-u onto G'-v fails to map v to u. Thus, the only reliable pseudosimilar pairs are those constructed as in Figure 3.

G':



<u>Figure 5</u>: Pseudosimilar cutpoints not suitable for the construction in Theorem 1.

3. <u>All vertices can be pseudosimilar!</u>

The construction in Theorem 1 provides k mutually pseudosimilar vertices among $3k^2-2k$ vertices. While k is free to increase, nevertheless, it forms a progressively smaller fraction of the total number of vertices in the graph. A relevant question is how many pseudosimilar vertices can be packed into a graph of fixed size. One of our early examples had n pseudosimilar pairs among 4n+1 vertices, so virtually half were pseudosimilar. We achieved various improvements until we ultimately found graphs having all their vertices paired by pseudosimilarity.

The general strategy for producing such graphs is motivated by the methods of Herndon and Ellzey [3] already mentioned above. Let H be any vertex symmetric graph with an arbitrary vertex labeled r. For each vertex v in G=H-r, there is an automorphism α of H mapping r to v. Then α^{-1} maps $\{v,r\}$ onto $\{r,\alpha^{-1}(r)\}$, and so $G-v = H-r-v \cong H-\alpha^{-1}(r)-r = G-\alpha^{-1}(r)$.
Thus, $\alpha^{-1}(r)$ and $v = \alpha(r)$ are removal-similar in G. They will be pseudosimilar if we can guarantee they are dissimilar. To prevent accidental similarity,

let us insist that G have the identity automorphism group. This requires H to have no nontrivial automorphisms keeping r fixed, for any such mapping could be restricted to G to yield an automorphism. Furthermore, H cannot have two automorphisms such that $\alpha(r) = \beta(r) = v$, for then $\beta^{-1}\alpha$ would fix r. Thus, for each vertex v in H there must be one and only one automorphism sending r to v. If we like, we may use this automorphism as a label for v. Of course, r is labeled by the identity.

We have forbidden G to have any automorphisms, yet $\alpha(r)$ and $\alpha^{-1}(r)$ could fail to be pseudosimilar if they just happened to be equal! This can happen if and only if some of the automorphisms of H include an orbit of length 2. Such automorphisms exist if and only if the order of the group is even. Consequently, we insist that the group (and so H) have odd order.

Finally, we notice that if the group $\Gamma(H)$ is abelian, the mapping $\Theta: G \to G$ defined by $\Theta(\alpha(r)) = \alpha^{-1}(r)$ is an automorphism of G. But G is required to have identity group. Therefore, $\Gamma(H)$ needs to be nonabelian.

To summarize the requirements of our construction, H must be simply vertex symmetric so that G will have identity group. Furthermore, $\Gamma(H)$ has to be an odd order nonabelian group.

We proceed to define a family of graphs in which many of the graphs have these properties. Let m and n be odd positive integers and suppose there exists an $a \not\equiv 1 \bmod n$ such that $a^m \equiv 1 \bmod n$ (this requires $\gcd(m,\phi(n)) \neq 1$). The graph H(m,n,a) has mn vertices labeled $v_{i,j}$ with i taken modulo m and j taken modulo n. For convenience we shall usually identify $v_{i,j}$ by simply listing the ordered pair of subscripts (i,j). We form H(m,n,a) by joining vertex (i,j) to vertices (i,k) for all $k \neq j$ as well as to (i+1,aj) and (i+1,aj+1). We visualize H(m,n,a) as having m cliques of size n arranged in

a circle with each vertex joined to two additional vertices in each neighboring clique. It is routine to verify that the mapping $\alpha(i,j) = (i+1,j)$ is an automorphism permuting the cliques in an m cycle. Also, the mapping $\beta(i,j) = (i,j+a^i)$ is well defined because $a^m \equiv a^0 \bmod n$ and β is an automorphism permuting the vertices within the cliques in n-cycles. Evidently, $\alpha^m = \beta^n = 1$ and $\beta\alpha = \alpha\beta^a$. Thus, α and β generate a nonabelian group of order mn, known as a normal or semidirect product of C_n by C_m. The automorphism $\alpha^i\beta^j$ maps (0,0) to (i,j) so all mn automorphisms are distinct, and H(m,n,a) is vertex transitive. If additional accidental automorphisms of H can be avoided, $G = H(m,n,a) - (0,0)$ has identity group. In G, vertex $(i,j) = \alpha^i\beta^j(0,0)$ is removal similar to $(\alpha^i\beta^j)^{-1}(0,0) = (-i,-ja^{-i})$. We note that (i,j) cannot equal $(-i,-ja^{-i})$, for otherwise $i \equiv -i \bmod n$ implies $i \equiv 0$ and then $j \equiv -j \bmod n$ implies $j \equiv 0$. Therefore (i,j) and $(-i,-ja^{-i})$ are pseudosimilar. These observations are summarized in the following theorem:

<u>Theorem 2</u>. If $G = H(m,n,a)-(0,0)$ has identity group, then each vertex (i,j) possesses a pseudosimilar mate $(-i,-ja^{-i})$.

<u>Lemma 3</u>. A sufficient condition for $G = H(m,n,a) - (0,0)$ to have identity group is that no non-empty subset of $\{1,a,a^2,\ldots,a^{m-1}\}$ sums to 0 modulo n.

<u>Proof</u>. Any automorphism of G induces an automorphism γ of H(m,n,a) fixing (0,0). The vertices (i,0) comprise an m-cycle traversing all the cliques. Call this cycle C. Now any i path from (0,0) to the i-th clique can only terminate at (i,0) or a vertex (i,j) for which a subset of $\{1,a,a^2,\ldots,a^{i-1}\}$ sums to j mod n. This is easily verified by induction, because in passing from the (i-1)-st clique to the i-th clique we multiply by a and add either 0 or 1. Thus, upon returning to the 0-th clique from the (m-1)-st clique, an m-cycle other than C is formed only if a non-empty subset of $\{1,a,a^2,\ldots,a^{m-1}\}$ sums to 0 mod n.

Since we forbade this, C is the unique m-cycle containing (0,0) and traversing all the cliques. Thus, γ must map C onto itself.

We wish to demonstrate that γ restricted to C must be the identity. If not, then clearly

(1) $\gamma(i,0) = (-i,0)$ for all i.

Now consider the cliques $\frac{m\pm1}{2}$. Since $\gamma(\frac{m-1}{2},0) = (\frac{m+1}{2},0)$, these two cliques must be interchanged by γ. These cliques are laced together by a 2n-cycle which includes the edge e from $(\frac{m-1}{2},0)$ to $(\frac{m+1}{2},0)$. Since γ inverts this edge onto itself, γ must yield an inversion of the entire 2n-cycle. Referring to the adjacencies between these two cliques, we trace this 2n-cycle in both directions to verify that

(2) $\gamma(\frac{m-1}{2},j) = (\frac{m+1}{2},-ja)$ for all j.

But notice that $(\frac{m-3}{2},0)$ is adjacent to both $(\frac{m-1}{2},0)$ and $(\frac{m-1}{2},1)$ while $(\frac{m+3}{2},0)$ is joined to $(\frac{m+1}{2},0)$ and $(\frac{m+1}{2},-a^{-1})$. Considering (1) and (2), we conclude that

(3) $\gamma(\frac{m-1}{2},1) = (\frac{m+1}{2},-a) = (\frac{m+1}{2},-a^{-1})$.

That is, $-a \equiv -a^{-1} \bmod n$. Thus $a^2 \equiv 1 \bmod n$. Since we already required $a^m \equiv 1 \bmod n$, and m is odd, we would have $a \equiv 1 \bmod n$, a contradiction. Hence, γ is the identity on C, namely $\gamma(i,0) = (i,0)$.

Finally, for each i, γ sends the 2n-cycle joining the i-th and (i+1)-st cliques onto itself keeping vertices (i,0) and (i+1,0) fixed. Therefore, γ is the identity on both cliques, and ultimately γ is the identity on all of H(m,n,a). Thus, the only automorphism of $G = H(m,n,a)-(0,0)$ is the identity. ∎

Remark 1. A partial converse of Lemma 3 is also true. If the entire sum $1 + a + a^2 + \dots + a^{m-1} = (a^m-1)/(a-1)$ is congruent to 0 modulo n, then the mapping $\delta(i,j) = (1,(a^i-1)/(a-1)-j)$ is well defined and easily seen to be an automorphism of G. Moreover, δ maps each vertex of clique 0 onto its removal

similar mate, making these pairs similar rather than pseudosimilar. This situation will arise whenever $\gcd(a-1,n) = 1$. Consequently, $\Gamma(H)$ cannot be the smallest odd ordered nonabelian group, of order 21, as this would require $m = 3$, $n = 7$, and $a = 2$ or 4.

Remark 2. We are more fortunate with the next possible order, 27. The lemma guarantees that $G = H(3,9,4)-(0,0)$ has identity group because $\{1,4,7\}$ has no nonempty subset that sums to 0 modulo 9. This graph has 13 pairs of pseudosimilar vertices.

More generally, the lemma guarantees that $G = H(m,a^m-1,a)-(0,0)$ will work for arbitrary odd $m \geq 3$ and arbitrary even $a \geq 4$, because any sum S is bounded by

$$(4) \quad 0 < S \leq 1 + a + a^2 + \ldots + a^{m-1} = (a^m-1)/(a-1) < a^m-1 .$$

Often odd values for a and smaller values for n can be used, provided one chooses n to be an odd divisor of a^m-1 and then verifies the forbidden sum property of the lemma.

Remark 3. What bearing does this construction have on the classical Reconstruction Conjecture? The fundamental building blocks used in creating nonreconstructable digraphs [5,6] are a family of tournaments T_n on $n = 2^k$ vertices, $k \geq 0$. The key property of these tournaments, in our present terminology, is that they have identity group yet for $k \geq 1$ all their vertices possess pseudo-similar mates. Two members T_m and T_n of this family can be joined with additional edges in various ways to produce several digraph pairs D and D' on $2^j + 2^k$ vertices such that for each pseudosimilar pair u and v in either T_m or T_n, we have $D-u \cong D'-v$. Thus D and D' are nonreconstructable digraphs.

Theorem 2 duplicates the crucial properties of T_n in graphs. One would hope that these properties could be exploited in building a pair of nonreconstructable graphs. The joining methods used for digraphs do not readily extend to these graphs, but some other technique might work.

On a more general level, the success of building graphs in which every vertex

is pseudosimilar to some other vertex should provide encouragement to those attempting to find nonreconstructable pairs. If all the vertices in a graph can be distinctive and yet be paired by pseudosimilarity, why should we not hope to find a pair of graphs with corresponding properties? Of course, since the smallest odd nonabelian group has order 21, the smallest graph we could hope to obtain by Theorem 2 has order 20. We suggest that any search for nonreconstructable graphs should begin at this size. This means an exhaustive search is out of the question.

At the very least, we feel we have identified the right neighborhood to be searched. Graphs having widely varying degrees are easily reconstructed because the vertices are so identifiable. Graphs that are nearly regular seem to provide maximal confusion. However, if one goes a step too far and considers regular graphs, reconstruction is trivial. The graphs of Theorem 2 have degrees of two consecutive values, say k and k+1. One can recognize the degree of the deleted vertex. This vertex was clearly joined to all vertices of degree k-1 in the subgraph and to none of those of degree k+1, but how can one decide which of the vertices of degree k are neighbors of the deleted vertex?

4. Related Problems

In addition to the Reconstruction Conjecture itself, our results suggest several related problems. We conclude with some comments on five of these questions.

Problem 1. Does a simply vertex transitive graph H exist for every odd ordered nonabelian group? For instance, our approach failed to yield an example for the smallest odd nonabelian group, which has order 21. Actually, we restricted ourselves considerably by requiring the m disjoint subgraphs to be cliques and by requiring consecutive cliques to be joined by only two edges at each vertex.

We chose this approach to facilitate the proof that H-(0,0) has identity group for a rather large class of graphs. All we really needed was cyclic symmetry, both permuting the "cliques" and within each "clique". The nonabelian group of order 21 is generated by $\alpha^3 = \beta^7 = 1$ and $\beta\alpha = \alpha\beta^2$. As before, we let the vertices of H be labeled (i,j) reading i mod 3 and j mod 7, and set $\alpha(i,j) = (i+1,j)$ and $\beta(i,j) = (i,j+2^i)$. The 21 automorphisms $\alpha^i\beta^j$ partition the edges of K_{21} into ten orbits of size 21. So long as we include or exclude full orbits, $\Gamma(H)$ will possess these 21 automorphisms. The trick is to select the right subset of orbits to forbid all other automorphisms. An exhaustive search uncovered four such graphs:

The first, H_1, has each vertex (i,j) joined to (i,j+1), (i+1,2j), (i+1,2j+1), and (i+1,2j+3). This choice of four orbits means H_1 is regular of degree 8. The next graph H_2 is obtained from H_1 by adding the edges joining (i,j) to (i,j+2) for every i and j. Detailed inspection is required to verify that the graphs $G_k = H_k - (0,0)$ have identity group as desired. The last two graphs, H_3 and H_4 are obtained by complementing the first two. Since H_2 and H_4 are both regular of degree 10, it was necessary to verify that these two are not isomorphic. Thus, we have constructed four graphs on 20 vertices, each having ten pseudosimilar pairs of vertices.

As the size of the nonabelian group increases, the number of orbits we have to work with goes up. This would seem to improve the chances for eliminating unwanted accidental automorphisms. However, the work of an exhaustive inspection soon becomes prohibitive. We conjecture that some graph exists for each odd ordered nonabelian group.

Problem 2. Are there any graphs having all vertices paired by pseudosimilarity which do not arise from nonabelian groups? If not, the four examples mentioned

in Problem 1 are the smallest possible. But if there are other ways to build such thorough pseudosimilarity, do any of these graphs have order less than 20?

Problem 3. Find the smallest graphs having k mutually pseudosimilar vertices. The construction of Theorem 1 achieved k pseudosimilar vertices out of $3k^2 - 2k$ vertices. For k=2, our construction degenerates to the Harary-Palmer graph of Figure 1 already found to be smallest [1]. We are inclined to suspect our construction is smallest possible, but we have no clue about how to prove it minimal.

Problem 4. Define two edges of a graph to be pseudosimilar if they are removal-similar and yet not similar. Harary and Palmer [2] gave examples of graphs having a single pseudosimilar pair of edges. Can graphs be found having k mutually pseudosimilar edges? Our only success in this area, the construction of graphs having pseudosimilar triples of edges, is described below. The approach does not seem to generalize to larger values of k.

Let T be the tournament on seven vertices, labeled modulo 7, with v_i dominating v_j if and only if $j-i \equiv 1, 2,$ or $4 \bmod 7$. The mappings $\alpha(v_i) = v_{i+1}$ and $\beta(v_i) = v_{2i}$ are automorphisms of T. Convert T to a graph H by replacing each arc v_iv_j with a subgraph $K_{1,3}$+e joining the unique endpoint at the tail of the arc and one of the vertices of degree two at the head. Graph H has 49 vertices, and retains the automorphisms possessed by T. To form G, add three new endpoints, u_0, u_1, and u_2 with each u_i joined to v_i. Clearly, any automorphism γ of G maps the set $\{v_1,v_2,v_3\}$ onto itself. But since these three vertices induce a transitive triple in T, they each are fixed points of γ. It is now easy to verify that each remaining v_j in turn is fixed, so γ is the identity. Thus, G has identity group.

Nevertheless, $G-u_2v_2 \cong G-u_0v_0$ by the automorphism corresponding to α in T, and similarly, $G-u_2v_2 \cong G-u_1v_1$ by the automorphism corresponding to β. Of course, $G-u_0v_0 \cong G-u_1v_1$ by the mapping corresponding to $\beta\alpha^{-1}$ Thus, G has a pseudosimilar triple of edges as claimed.

Problem 5. Can graphs be found in which all edges are paired by pseudosimilarity? Our rather firm opinion is no. If we are correct, it may be that the differences in pseudosimilarity for vertices and edges create fundamental differences in the vertex Reconstruction Conjecture and the edge Reconstruction Conjecture. We would not be surprised to learn, eventually, that the edge conjecture is true but the vertex conjecture is false!

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13. ABSTRACT

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Graphs						
Reconstruction Conjecture						
Automorphism Groups						

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